Entangling Two-Atom Through Cooperative Interaction Under Stimulated Emission

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Abstract

We discuss the generation of two-atom entanglement inside a resonant microcavity under stimulated emission (STE) interaction. The amount of entanglement is shown based on different atomic initial state. For each kind of intial state, we obtain the entanglement period and the entanglement critical point, which are found to deeply depend on driving field density. In case of atomic state $|ee\rangle$, the entanglement can be induced due to STE. In case of atomic state $|eg\rangle$, there is a competition between driving field indued entanglement and STE induced entanglement. When two atoms are initially in $|gg\rangle$, we can find a lumbar region where STE increases the amount and period of entanglement.

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1 Introduction

One of the most interesting features of quantum mechanics is the correlation between pairwise-entangled quantum states of two spatially separated particles, which is called EPR pairs. Besides the applications of EPR pairs on investigating the conceptual foundations of quantum mechanics, such as testing the violation of Bell inequality, a great deal of interesting has been intensively focused on designing and realizing possible quantum entangling proposals that can

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be essential ingredients in quantum communication [1, 2, 3] and quantum computation [4]. These EPR pairs can be formatted in different physical systems such as trapped ions [5], spins in nuclear magnetic resonance [6], superconductor Josephsen junctions [7], Cooper pairs in solid states quantum-dots [8], and cavity quantum-electrodynamics systems (CQED) [9]. Among these systems, CQED system has been deeply studied for entangling two atoms or two modes field in constructing quantum logic gates [10, 11] or quantum memory [12]. Alternatively, two atoms can be entangled through contineously driving by a coherent pump field [11, 13], the assistance of a themal cavity field [14, 15, 16], or even the inducement from the atomic spontaneous emission [15, 17]. Generally, these schemes are effective since there are some kinds of interaction between atom and atom or atom and cavity field. While, there is another interaction that is uaually not included: the stimulated emission (STE, which refers to Einstein B coefficient [18]) of atoms in a atomic ensemble. The STE of atom emerges when atom in higher energy level is driven by a polarized photon [19]. Especially when atomic population inversion is realized, in a laser system, STE plays a key role in photon absorb-emission process. In Ref.[20], taking into account the STE, the authors study the resonance fluorescence spectrum and present five peaks are formed due to STE. In solving the resonance fluorescent spectrum, the authors treat the emited photon as a new driving field acting on both atoms since the emited photon has the identical character with that of driving photon. In this paper, we consider a system with this interaction and try to analize the entanglement character of two-atom. This system is discribed in the first section. In second section, the measurement of entanglement is presented. And in last section, some results on two-atom entanglement nature are obtained.

2 Cooperative Interaction Between atoms

We consider a system constituted by two two-level atoms located in a nanocavity and a single mode cavity field. Figure 1 shows the schematic diagram of this system.

The model is discribed as follow: The cavity is isolated from its surroundings.

Both identical atoms have two internal energy levels: an excited states $|e\rangle$ and a ground state $|g\rangle$. Either atom can, when it is excited to state $|e\rangle$, transit to state $|g\rangle$ under the driving of an external photonic field with frequency equals the difference of energy levels $|e\rangle$ and $|g\rangle$ and emit a polarized photon which is identical with the driving one. Also, either atom can absorb such polarized photon when it is in state $|g\rangle$ and jump to state $|e\rangle$. The cavity wall is arranged to be mirrors, so that the emitted photon can be fully reflected and finally be absorbed by the atoms. That is, besides driving photonic field E, the emited polarized photon from atom 1 can also act as a new driving photonic field E' with repect to atom 2, and *vice versa*, as if the atoms exchange a photon between them owing to STE, this is a kind of cooperative interaction. As a result, the whole cavity field is the sum of two fields E and E'. The Hamiltonian of the system in the interaction picture reads [20]

$$H = g_{drv} \sum_i (a\sigma_i^+ + a^+\sigma_i^-) + g_{stm} \sum_{i<>j} \left[\sigma_i^z (a\sigma_j^+ + a^+\sigma_j^-) + (a\sigma_j^+ + a^+\sigma_j^-)\sigma_i^z\right] \tag{1}$$

where a, a^+ are eliminate and create operators of driving field, $\sigma_i^+ = |e\rangle_i \langle g|$, $\sigma_i^- = |g\rangle_i \langle e|$ are transition operators of atom i and $\sigma_i^z = \frac{1}{2}(|e\rangle_i \langle e| - |g\rangle_i \langle g|)$ is the inversion operator of atom i, g_{drv} and g_{stm} represent the coupling strength between atomic transition $|e\rangle \longleftrightarrow |g\rangle$ and field E or E' respectively. Generally, g_{stm} is determined by STE coefficient, atomic density and g_{drv}. For convenience, we simply call g_{stm} as STE coupling strength. The second sum for subscripts i and j proceeds for $i, j = 1, 2$ and $i \neq j$. In the following analysis, we will investigate the entanglement nature of two-atom sub-system evoluted state under this interaction.

The evolution of the density matrix of global system with initial state $\rho(0)$ is controlled by an unitary operator $\hat{U}(t) = e^{-iHt/\hbar}$, formally, it is $\rho(t) = \hat{U}(t)\rho(0)\hat{U}^\dagger(t)$. Under the above assumption, two two-level atoms form a $2 \otimes 2$ dimensional Hilbert subspace as

$$(H)_a = (H_1)_a \otimes (H_2)_a = \begin{pmatrix} |e\rangle_1 \langle e| & |e\rangle_1 \langle g| \\ |g\rangle_1 \langle e| & |g\rangle_1 \langle g| \end{pmatrix} \otimes \begin{pmatrix} |e\rangle_2 \langle e| & |e\rangle_2 \langle g| \\ |g\rangle_2 \langle e| & |g\rangle_2 \langle g| \end{pmatrix}. \tag{2}$$

By expanding $\hat{U}(t)$ into a combination of Taylor series, we rewrite the evolution operator matrix in the basis of Equ.2 as following analytical form

$$\hat{U}(t) = \begin{pmatrix} 2g^2 a\Theta a^+ + 1 & -iga\Phi & -iga\Phi & 2g^2\gamma a\Theta a \\ -ig\frac{\sin\Omega t}{\Omega}a^+ & \frac{1}{2}(\cos\Omega t + 1) & \frac{1}{2}(\cos\Omega t - 1) & -ig\gamma\Phi a \\ -ig\frac{\sin\Omega t}{\Omega}a^+ & \frac{1}{2}(\cos\Omega t - 1) & \frac{1}{2}(\cos\Omega t + 1) & -ig\gamma\Phi a \\ 2g^2\gamma a^+\Theta a^+ & -ig\gamma a^+\Phi & -ig\gamma a^+\Phi & 2g^2\gamma^2 a^+\Theta a + 1 \end{pmatrix} \tag{3}$$

, where, we have set $\Theta = \frac{\cos\Omega t - 1}{\Omega}$, $\Phi = \frac{\sin\Omega t}{\Omega}$, $\Omega = \{2g^2[(\gamma^2+1)a^+a + \gamma^2]\}^{\frac{1}{2}}$, $g = g_{drv} + g_{stm}$ and $\gamma = \frac{g_{drv} - g_{stm}}{g_{drv} + g_{stm}}$. Note that only when $\gamma = 1$ does this matrix equal that in Ref.[14]. This is obvious because $\gamma = 1$ corresponds to a system without STE.

What attracts us is the evolution process of the two-atom sub-system density matrix which is obtained by tracing over the field variables of system density matrix $\rho(t)$. We can expect the cooperative interaction can induce atom-atom entanglement during the evolution.

3 Measurement of entanglement degree

Whatever be the initial state of two-atom, the time evolution operator $\hat{U}(t)$ would reduce most of the off-diagonal elements of the density matrix. The resulting two-atom density matrix can be written as

$$\rho(t) = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & E & 0 \\ 0 & E & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}. \tag{4}$$

Using the entanglement degree defined by Wootters concurrence simplified from the entanglement of formation [21]

$$C(\rho) = \max\{0, 2\max\{\lambda_i\} - \sum_i \lambda_i\} \tag{5}$$

where λ_i are the four non-negative square roots of the eigenvalues of the non-Hermitian matrix $\rho(t)\tilde{\rho}(t)$ with $\tilde{\rho}(t) = (\sigma_y \otimes \sigma_y)\rho^*(t)(\sigma_y \otimes \sigma_y)$. We obtain

$$\lambda_1 = \lambda_2 = \sqrt{A \cdot D},\ \lambda_3 = E + \sqrt{B \cdot C},\ \lambda_4 = \left|E - \sqrt{B \cdot C}\right|. \tag{6}$$

So, the Concurrence of the density must be $C(\rho) = \{0,\ 2(\min\{E, \sqrt{B \cdot C}\} - \sqrt{A \cdot D})\}$. Then, $\min\{E, B \cdot C\} > A \cdot D$ is the sufficient and necessary condition for emerging two-atom entanglement. Under this circumstance, the entanglement degree of two-atom subsystem is

$$C(\rho) = 2(\min\{E, \sqrt{B \cdot C}\} - \sqrt{A \cdot D}). \tag{7}$$

Alternatively, the two-atom entanglement can be measured by the criteria defined as the patial transposition proposed by Peres and Horodecki which is written as $\varepsilon = -2 \sum_i \mu_i$ with μ_i corresponding to the negative eigenvalues of the partial trasposition $\rho_a^T(t)$ of density matrix. It has been discussed in Ref. [16] that only when $E^2 > A \cdot D$ may the entanglement of two-atom be created. This criteria for entanglement is equivalent with that of Wootters Concurrence when $E^2 \leqslant B \cdot C$ (in fact, the equality is obvious in the following results). Both criterias are suitable for measuring the entanglement of arbitrary two qubits system whether the system being pure state or mixed one.

Here, we use Concurrence as the entanglement measurement.

4 Atom-Atom entanglement discussion

We assume the initial cavity driving field is in single mode Fork state $|n\rangle_f$ with photon number n presenting the cavity field density. So that the reduced two-atom sub-system density matrix is

$$\rho_a(t) = Tr_f \rho(t) = Tr_f \left[\hat{U}(t)\rho(0)\hat{U}^\dagger(t)\right] = \sum_n {}_f\langle n|\, \hat{U}(t)\, |0\rangle_f\, \rho_a(0)_f\, \langle 0|\, \hat{U}^\dagger(t)\, |n\rangle_f \tag{8}$$

where $\rho_a(0)$ is the initial state of two-atom subsystem. The matrix element ${}_f\langle n|\, \hat{U}(t)\, |0\rangle_f$ presents the influence of atomic transition on the cavity mode. In the following analysis, we will present the entanglement nature of two atoms under STE when they are initially in different state. For convencience, we set $g_{drv} \equiv 1$ in following analysis.

Firstly, we consider two atoms are both initialy in their excited state, that is

$\rho_a(0) = |e\rangle_{11} \langle e| \otimes |e\rangle_{22} \langle e|$. We get, in Equ.5, $A = |U_{11}|^2 = [1+2(n+1)\frac{\cos g\xi t-1}{\xi^2}]^2$, $B = C = E = |U_{21}|^2 = (n+1)\frac{\sin^2 g\xi t}{\xi^2}$, $D = |U_{41}|^2 = 4\gamma^2(n+1)(n+2)\frac{(\cos g\xi t-1)^2}{\xi^4}$, where $\xi = \sqrt{2[(\gamma^2+1)(n+1)+\gamma^2]}$. It has been be stressed in Ref. [14] that no two-atom entanglement can be generated when two atoms are initially in $|e\rangle_1 |e\rangle_2$ no matter what state the cavity state is. While, when STE is included, the result is apparently different. We can find that the necessary and sufficient condition of generating positive concurrence in Equ.8 is $0 \leqslant \gamma < \sqrt{\frac{n+1}{n+2}}$. That is, there exists a critical point $\gamma_0 = \sqrt{\frac{n+1}{n+2}}$ ($g_{ste,crit} = \left(\sqrt{n+2} - \sqrt{n+1}\right)^2 g_{drv}$) which turns out to be the minimum value of STE coefficient for generating two-atom entanglement that are initially in excited states. One of the ways to decrease the critical point is increasing the density of field. Extremely, for large n, this point tends to zero, which means, in a high field density cavity, even a slight STE can induce two-atom entanglement. For a vacuum field, this point is about $0.17 g_{drv}$. To show these properties, we plot two-atom entanglement as a founction of Time-t and γ in Figure 1a and Figure 1b with different driving field density n. Both Figures present that the Concurrence is almost a monotone decreasing founction of γ. Along t axis, the Concurrence presents a sine-quare-like behavior with periodical maximum and minimum-zero. While, this period can be changed by alternating γ. Only when $g_{ste} = g_{drv}$ does the Concurrence act as an exactly sine-quare founction $\sin^2 \sqrt{2(n+1)}t$, where the period presents as $\frac{\pi}{\sqrt{2(n+1)}}$ which is, for example, exactly $\frac{\pi}{2}$ for $n = 0$ and $\frac{\sqrt{2}\pi}{4}$ for $n = 1$. Elsewhere, the period of entanglement along t axis is $\frac{2\pi}{\xi}$ for a given γ. It is fascinating that the driving field density n not only determines the critical value of g_{stm}, but also takes great influence on the entanglement-disentanglement period. Generally, the larger the driving field density the smaller the entanglement period. Another unapparent character of two Figures is the peak of the Concurrence for a same γ can be increased by increasing field density except for the range of γ very close to zero. This can be easily understood since $g_{stm,crit}$ can be increased by increasing n. In constructing practical quantum logical gates, we may need strong and long sustained entanglement, thus, we should make a suitable choice of controllable physical parameters such as the STE coefficient and the density

of monochromatic driving field.

Secondly, we assume one of the atoms is initially excited and the other has falled to its ground state, thus the two-atom sub-system initial state is $\rho_s(0) = |e\rangle_{11}\langle e| \otimes |g\rangle_{22}\langle g|$. The resulting two-atom density matrix elements can be obtained as $A = |U_{12}|^2 = n\frac{\sin^2 g\xi t}{\xi^2}$, $B = |U_{22}|^2 = (\cos g\xi t + 1)^2/4$, $C = |U_{23}|^2 = (\cos g\xi t - 1)^2/4$, $D = |U_{42}|^2 = \gamma^2(n+1)\frac{\sin^2 g\xi t}{\xi^2}$, $E = |U_{22}U_{23}| = \sin^2 g\xi t/4$, where $\xi = \sqrt{2[(\gamma^2+1)n+\gamma^2]}$. We can also find the necessary and sufficient condition of generating positive entanglement is $\gamma \neq \sqrt{\frac{n}{n+1}}$. The critical point is $\gamma_0 = \sqrt{\frac{n}{n+1}}$ ($g_{ste,crit} = \left(\sqrt{n+1} - \sqrt{n}\right)^2 g_{drv}$) which can be shifted towards zero by increasing n (we will show the reason why $n > 0$ must be satisfied in this situation). The entanglement situation is shown in Figure 2a, 2b, 2c. The whole area can be seperated into two regions: the region where $0 \leqslant \gamma < \gamma_0$ and the region where $\gamma_0 < \gamma \leqslant 1$. In the first region, the entanglement increases rapidly with respect to γ. Especially, in the area that $\gamma \to 0$, where two coupling strengthes g_{drv} and g_{stm} are comparable, the entanglement monotonously reaches its peak. While, it can be easily proved that this peak can never exceed 0.5 which is the most probable maximum value of entanglement. For a given γ, the time evolution of entanglement presents periodical loss and revival with a period $\frac{\pi}{\xi}$. Obviously, to obtain long time sustained entanglement, STE should be outstanding and driving field density should not be large. When $n \to 0$ and $\gamma \to 0$, the period tends to *infinite*! Under this circumstance, large entanglement can never be obtained in finite time. In the second region, where STE is very weak, the coupling between atom and driving field is dominating. The result is similar with that obtained in Ref.[14]. But the resulting entanglement is much weak (see Figure 2a) and only emerges when driving field density is small (see Figure 2b). In both regions, the loss and revival of the entanglement also shows to be periodical. When n tends to zero, the period, which is approximately $\frac{\pi}{\sqrt{2}\gamma}$, is only dependent on difference between two coupling strengthes γ. It should be stressed that when driving field density is large, and the first-term interaction in Equ. 2 can hardly induce entanglement, STE can still generate entanglement. To sum up, we see there is a competition, which depends on n and γ, between first-term-interaction and

second-term-interaction in Equ. 2. The competition behaves with: which is the domination of entanglement, how about the shift of the critical point and what is the contrast of entanglement periods in two regions. While, whatever be the competition, apparent STE can enhance two-atom entanglement.

5 Conclusion

We have discussed the generation of two-atom entanglement inside a resonant microcavity under an auxiliary interaction-STE. The entanglement when two atoms are initially in $|e\rangle_1 |e\rangle_2$ and $|e\rangle_1 |g\rangle_2$ is studied. Some meaningful results are obtained with the assistance of different STE. The obtained entanglement is simply determined by a analytic sine-like function of time and difference between two coefficients. In both cases, we obtained the critical points of generating entanglement as well as the entanglement peroid. These two quantities can both be controlled by adjusting the density of driving field or stimulated emission coefficient in experiment scenario. We have created two-atom entanglement in first case where there is shown impossible generating entanglement without STE. In second case, we found a competition between the interaction with and without STE, while, the amplitude of entanglement with STE is much larger than that without STE, so is the period. In third case, a lumbar region is found through raising STE coefficient. This region is some entanglement place corresponding to a certain time interval where entanglment can hardly be genenrated without STE.

While, in dealing with this system, we do not take into account the atomic spontaneous emission which leads to a line width of the emitted photon. Also, we do not inlude the dissipation of the cavity that has been considered in many papers [13, 22]. Even after including these effects, the STE could still plays an important role in a collectively excited atomic ensemble. The entanglement induced by stimulated emission should also be considered when using atoms to construct quantum logic gate or storage of photon information. Though we only studied two atoms in a large number of an atomic ensemble, the results can be extended to a multi-atom system.

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Figure Captions

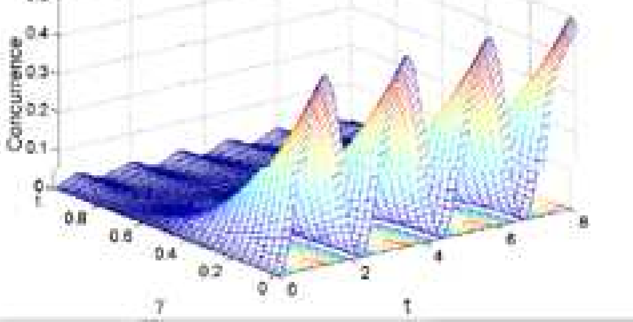
Figure 1: Schematic diagram for two two-level atoms system cooperative initeraction through STE. Under the drive of a photonic field E, one excited atom can fall to its lower state. The emitted photon acts as a new field E'. Another atom may be excited by E' and jump to its higher state. And vice versar. Then they complete a process of cooperative interaction.

Figure 2a: Two-atom (initially in $|e,e\rangle$) entanglement as a founction of t and γ for $n=1$.

Figure 2b: Two-atom (initially in $|e, e\rangle$) entanglement as a founction of t and γ for $n = 3$.

Figure 3a: Two-atom (initially in $|e, g\rangle$) entanglement as a founction of t and γ for $n = 3$.

Figure 3b: Two-atom (initially in $|e, g\rangle$) entanglement as a founction of t and γ for $n = 1$.



Concurrence
0.5
0.4
0.3
0.2
0.1
0
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
t